

# GROUPS OF ORDER 16

## SUMMARY

order	16
# abelian	5
# other decomposable	2
# other indecomposable*	7
<b>TOTAL</b>	<b>14</b>

$$16 = \langle A^{16} \rangle \quad \theta = e^{2\pi i/16}$$

elts	1	A <sup>8</sup>	A <sup>4</sup>	A <sup>12</sup>	A <sup>2</sup>	A <sup>14</sup>	A <sup>6</sup>	A <sup>10</sup>	A	A <sup>15</sup>	A <sup>3</sup>	A <sup>13</sup>	A <sup>5</sup>	A <sup>11</sup>	A <sup>7</sup>	A <sup>9</sup>	ℳ
CC	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
χ <sup>1</sup>	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
χ <sup>2</sup>	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1
χ <sup>3</sup>	1	1	1	1	-1	-1	-1	-1	i	-i	-i	i	i	-i	-i	i	2
χ <sup>4</sup>	1	1	1	1	-1	-1	-1	-1	-i	i	i	-i	-i	i	i	-i	2
χ <sup>5</sup>	1	1	-1	-1	i	-i	θ <sup>2</sup>	θ <sup>2</sup>	θ <sup>2</sup>	-iθ <sup>2</sup>	θ <sup>6</sup>	-θ <sup>2</sup>	-θ <sup>2</sup>	θ <sup>6</sup>	-iθ <sup>2</sup>	θ <sup>2</sup>	3
χ <sup>6</sup>	1	1	-1	-1	-i	i	θ <sup>2</sup>	θ <sup>2</sup>	-iθ <sup>2</sup>	θ <sup>2</sup>	-θ <sup>2</sup>	iθ <sup>2</sup>	iθ <sup>2</sup>	-θ <sup>2</sup>	θ <sup>2</sup>	-iθ <sup>2</sup>	3
χ <sup>7</sup>	1	1	-1	-1	i	-i	θ <sup>2</sup>	θ <sup>2</sup>	-θ <sup>2</sup>	iθ <sup>2</sup>	-iθ <sup>2</sup>	θ <sup>2</sup>	θ <sup>2</sup>	-iθ <sup>2</sup>	iθ <sup>2</sup>	-θ <sup>2</sup>	3
χ <sup>8</sup>	1	1	-1	-1	-i	i	θ <sup>2</sup>	θ <sup>2</sup>	iθ <sup>2</sup>	-θ <sup>2</sup>	θ <sup>2</sup>	-iθ <sup>2</sup>	-iθ <sup>2</sup>	θ <sup>2</sup>	-θ <sup>2</sup>	iθ <sup>2</sup>	3
χ <sup>9</sup>	1	-1	i	-i	θ <sup>2</sup>	-iθ <sup>2</sup>	θ <sup>2</sup>	θ <sup>2</sup>	θ	-iθ <sup>3</sup>	θ <sup>3</sup>	-iθ	iθ	-θ <sup>3</sup>	iθ <sup>3</sup>	-θ	0
χ <sup>10</sup>	1	-1	-i	i	-iθ <sup>2</sup>	θ <sup>2</sup>	θ <sup>2</sup>	θ <sup>2</sup>	-iθ <sup>3</sup>	θ	-iθ	θ <sup>3</sup>	-θ <sup>3</sup>	iθ	-θ	iθ <sup>3</sup>	0
χ <sup>11</sup>	1	-1	i	-i	θ <sup>2</sup>	-iθ <sup>2</sup>	θ <sup>2</sup>	θ <sup>2</sup>	-θ	iθ <sup>3</sup>	-θ <sup>3</sup>	iθ	-iθ	θ <sup>3</sup>	-iθ <sup>2</sup>	θ	0
χ <sup>12</sup>	1	-1	-i	i	-iθ <sup>2</sup>	θ <sup>2</sup>	θ <sup>2</sup>	θ <sup>2</sup>	iθ <sup>3</sup>	-θ	iθ	-θ <sup>3</sup>	θ <sup>3</sup>	-iθ	θ	-iθ <sup>2</sup>	0
χ <sup>13</sup>	1	-1	i	-i	-θ <sup>2</sup>	iθ <sup>2</sup>	θ <sup>2</sup>	θ <sup>2</sup>	iθ	-θ <sup>3</sup>	-iθ <sup>3</sup>	θ	θ <sup>9</sup>	iθ <sup>3</sup>	θ <sup>3</sup>	-iθ	0
χ <sup>14</sup>	1	-1	-i	i	iθ <sup>2</sup>	-θ <sup>2</sup>	θ <sup>2</sup>	θ <sup>2</sup>	θ <sup>3</sup>	-iθ	-θ	iθ <sup>3</sup>	-iθ <sup>3</sup>	θ	θ <sup>5</sup>	-θ <sup>3</sup>	0
χ <sup>15</sup>	1	-1	i	-i	-θ <sup>2</sup>	iθ <sup>2</sup>	θ <sup>2</sup>	θ <sup>2</sup>	-iθ	θ <sup>3</sup>	iθ <sup>3</sup>	-θ	θ	-iθ <sup>3</sup>	-θ <sup>3</sup>	iθ	0
χ <sup>16</sup>	1	-1	-i	i	iθ <sup>2</sup>	-θ <sup>2</sup>	θ <sup>2</sup>	θ <sup>2</sup>	-θ <sup>3</sup>	iθ	θ	-iθ <sup>3</sup>	iθ <sup>3</sup>	-θ	-iθ	θ <sup>3</sup>	0
order	1	2	4	4	8	8	8	8	16	16	16	16	16	16	16	16	

### Normal subgroups

Classes	H	G/H	∩
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	8	2
2	1 + 2 + 3 + 4	4	4
3	1 + 2	2	8

1   Φ  
2  
3

Maximal subgroups: [8]

$$16.02 = \langle A^8, B^2 \rangle = 8 \times 2 \quad \theta = e^{\pi i/4}$$

CC	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	$\mathfrak{K}$
$\chi_1$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
$\chi_2$	1	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1
$\chi_3$	1	1	-1	-1	i	-i	-i	i	1	1	-1	-1	i	-i	-i	i	2
$\chi_4$	1	1	-1	-1	-i	I	i	-i	1	1	-1	-1	-i	i	i	-i	2
$\chi_5$	1	-1	I	-i	$\theta$	$\theta^7$	$\theta^3$	$\theta^5$	1	-1	i	-i	$\theta$	$\theta^7$	$\theta^3$	$\theta^5$	3
$\chi_6$	1	-1	-i	i	$\theta^7$	$\theta$	$\theta^5$	$\theta^3$	1	-1	-i	i	$\theta^7$	$\theta$	$\theta^5$	$\theta^3$	3
$\chi_7$	1	-1	-i	i	$\theta^3$	$\theta^5$	$\theta$	$\theta^7$	1	-1	-i	i	$\theta^3$	$\theta^5$	$\theta$	$\theta^7$	3
$\chi_8$	1	-1	I	-i	$\theta^5$	$\theta^3$	$\theta^7$	$\theta$	1	-1	i	-i	$\theta^5$	$\theta^3$	$\theta^7$	$\theta$	3
$\chi_9$	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	4
$\chi_{10}$	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	5
$\chi_{11}$	1	1	-1	-1	i	-i	-i	i	-1	-1	1	1	-i	i	i	-i	6
$\chi_{12}$	1	1	-1	-1	-i	I	i	-i	-1	-1	1	1	i	-i	-i	i	6
$\chi_{13}$	1	-1	I	-i	$\theta$	$\theta^7$	$\theta^3$	$\theta^5$	-1	1	-i	i	$-\theta$	$-\theta^7$	$-\theta^3$	$-\theta^5$	7
$\chi_{14}$	1	-1	-i	i	$\theta^7$	$\theta$	$\theta^5$	$\theta^3$	-1	1	i	-i	$-\theta^7$	$-\theta$	$-\theta^5$	$-\theta^3$	7
$\chi_{15}$	1	-1	-i	i	$\theta^3$	$\theta^5$	$\theta$	$\theta^7$	-1	1	i	-i	$-\theta^3$	$-\theta^5$	$-\theta$	$-\theta^7$	7
$\chi_{16}$	1	-1	I	-i	$\theta^5$	$\theta^3$	$\theta^7$	$\theta$	-1	1	-i	i	$-\theta^5$	$-\theta^3$	$-\theta^7$	$-\theta$	7
order	1	2	4	4	8	8	8	8	2	2	4	4	8	8	8	8	

### Normal subgroups

Classes	H	G/H	$\cap$
1 + 2 + 3 + 4 + 9 + 10 + 11 + 12	$4 \times 2$	2	1
1 + 2 + 9 + 10	$2^2$	4	2
1 + 9	2	8	3
1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	8	2	4
1 + 2 + 3 + 4 + 13 + 14 + 15 + 16	8	2	5
1 + 2 + 11 + 12	4	$2^2$	6
1 + 10	2	8	7
1 + 2 + 3 + 4	4	$2^2$	$1 \cap 4 \quad \Phi$
1 + 2	2	$4 \times 2$	$2 \cap 4$

Maximal subgroups:  $[4 \times 2]$ ,  $[8] \times 2$

$$16.03 = \langle A^4, B^4 \rangle = 4^2$$

CC	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	$\mathcal{K}$
$\chi_1$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
$\chi_2$	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1
$\chi_3$	1	-1	i	-i	1	-1	i	-i	1	-1	i	-i	1	-1	i	-i	2
$\chi_4$	1	-1	-i	i	1	-1	-i	i	1	-1	-i	i	1	-1	-i	i	2
$\chi_5$	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	3
$\chi_6$	1	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	4
$\chi_7$	1	-1	i	-i	1	-1	i	-i	-1	1	-i	i	-1	1	-i	i	5
$\chi_8$	1	-1	-i	i	1	-1	-i	i	-1	1	i	-i	-1	1	i	-i	5
$\chi_9$	1	1	1	1	-1	-1	-1	-1	i	i	i	i	-i	-i	-i	-i	6
$\chi_{10}$	1	1	-1	-1	-1	-1	1	1	i	i	-i	-i	-i	-i	i	i	7
$\chi_{11}$	1	-1	i	-i	-1	1	-i	i	i	-i	-1	1	-i	i	1	-1	8
$\chi_{12}$	1	-1	-i	i	-1	1	i	-i	i	-i	1	-1	-i	i	-1	1	9
$\chi_{13}$	1	1	1	1	-1	-1	-1	-1	-i	-i	-i	-i	i	i	i	i	6
$\chi_{14}$	1	1	-1	-1	-1	-1	1	1	-i	-i	i	i	i	i	-i	-i	7
$\chi_{15}$	1	-1	i	-i	-1	1	-i	i	-i	i	1	-1	i	-i	-1	1	9
$\chi_{16}$	1	-1	-i	i	-1	1	i	-i	-i	i	-1	1	i	-i	1	-1	8
order	1	2	4	4	2	2	4	4	4	4	4	4	4	4	4	4	

### Normal Subgroups

	Classes	H	G/H	$\cap$	
1	1 + 2 + 5 + 6 + 9 + 10 + 13 + 14	$4 \times 2$	2	1	
2	1 + 5 + 9 + 13	4	4	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	$4 \times 2$	2	3	
4	1 + 2 + 5 + 6 + 11 + 12 + 15 + 16	$4 \times 2$	2	4	
5	1 + 5 + 10 + 14	4	4	5	
6	1 + 2 + 3 + 4	4	4	6	
7	1 + 2 + 7 + 8	4	4	7	
8	1 + 6 + 11 + 16	4	4	8	
9	1 + 6 + 11 + 16	4	4	9	
9	1 + 2 + 5 + 6	$2^2$	$2^2$	$1 \cap 3$	$\Phi$
10	1 + 2	2	$4 \times 2$	$1 \cap 6$	
11	1 + 6	2	$4 \times 2$	$1 \cap 8$	<b>Maximal subgroups:</b>
12	1 + 5	2	$4 \times 2$	$2 \cap 3$	$[4 \times 2] \times 3$

$$16.04 = \langle A^4, B^2, C^2 \rangle = 4 \times 2^2$$

CC	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	$\mathfrak{K}$
$\chi_1$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
$\chi_2$	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1
$\chi_3$	1	-1	i	-i	1	-1	i	-i	1	-1	i	-i	1	-1	i	-i	2
$\chi_4$	1	-1	-i	i	1	-1	-i	i	1	-1	-i	i	1	-1	-i	i	2
$\chi_5$	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	3
$\chi_6$	1	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	4
$\chi_7$	1	-1	i	-i	1	-1	i	-i	-1	1	-i	i	-1	1	-i	i	5
$\chi_8$	1	-1	-i	i	1	-1	-i	i	-1	1	i	-i	-1	1	i	-i	5
$\chi_9$	1	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	6
$\chi_{10}$	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1	7
$\chi_{11}$	1	-1	i	-i	-1	1	-i	i	1	-1	i	-i	-1	1	-i	i	8
$\chi_{12}$	1	-1	-i	i	-1	1	i	-i	1	-1	-i	i	-1	1	i	-i	8
$\chi_{13}$	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	9
$\chi_{14}$	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	-1	10
$\chi_{15}$	1	-1	i	-i	-1	1	-i	i	-1	1	-i	i	1	-1	i	-i	11
$\chi_{16}$	1	-1	-i	i	-1	1	i	-i	-1	1	i	-i	1	-1	-i	i	11
order	1	2	4	4	2	2	4	4	2	2	4	4	2	2	4	4	

### Normal subgroups

Classes	H	G/H	$\cap$	Classes	H	G/H	$\cap$	
1	1+2+5+6+9+10+13+14	$2^3$	2	15	1 + 5	2	$4 \times 2$	$2 \cap 3$
2	1 + 5 + 9 + 13	$2^2$	4	16	1 + 9	2	$4 \times 2$	$2 \cap 6$
3	1+2+3+4+5+6+7+8	$4 \times 2$	2	17	1 + 13	2	$4 \times 2$	$2 \cap 10$
4	1+2+5+6+11+12+15+16	$4 \times 2$	2	18	1 + 2 + 3 + 4	4	$2^2$	$3 \cap 6$
5	1 + 5 + 10 + 14	$2^2$	4	19	1 + 2 + 7 + 8	4	$2^2$	$3 \cap 7$
6	1+2+3+4+9+10+11+12	$4 \times 2$	2	20	1 + 6	2	$4 \times 2$	$3 \cap 8$
7	1+2+7+8+9+10+15+16	$4 \times 2$	2	21	1+2+11+12	4	$2^2$	$4 \cap 6$
8	1 + 6 + 9 + 14	$2^2$	4	22	1+2+15+16	4	$2^2$	$4 \cap 7$
9	1+2+3+4+13+14+15+16	$4 \times 2$	2	23	1 + 10	2	$4 \times 2$	$5 \cap 6$
10	1+2+7+8+11+12+13+14	$4 \times 2$	2	24	1 + 14	2	$4 \times 2$	$5 \cap 8$
11	1 + 6 + 10 + 13	$2^2$	4	25	1 + 2	2	$2^3$	$12 \cap 13$
12	1 + 2 + 5 + 6	$2^2$	$2^2$					
13	1 + 2 + 9 + 10	$2^2$	$2^2$					
14	1 + 2 + 13 + 14	$2^2$	$2^2$					

$$\mathfrak{K} = 25$$

Max subgroups:  $[2^3], [4 \times 2] \times 6$

$$16.05 = \langle A^2, B^2, C^2, D^2 \rangle = 2^2 \times 2^2$$

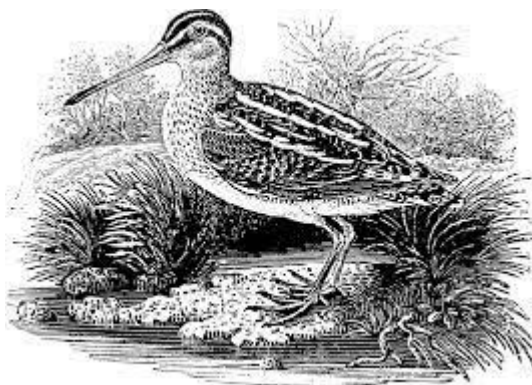
CC	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	$\mathfrak{K}$
$\chi_1$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
$\chi_2$	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1
$\chi_3$	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	2
$\chi_4$	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	3
$\chi_5$	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	4
$\chi_6$	1	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	5
$\chi_7$	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	6
$\chi_8$	1	-1	-1	1	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	7
$\chi_9$	1	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	8
$\chi_{10}$	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1	9
$\chi_{11}$	1	-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	10
$\chi_{12}$	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	1	-1	11
$\chi_{13}$	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	12
$\chi_{14}$	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	-1	13
$\chi_{15}$	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1	14
$\chi_{16}$	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1	15
order	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	

There are too many subgroups to list. Here is a summary:

order	#	H	G/H
8	15	$2^3$	2
4	35	$2^2$	$2^2$
2	15	2	$2^3$

### BEWICK'S WOODCUTS

In order to keep each group on a single page, or pair of pages, there is going to be a certain amount of blank space. Thomas Bewick [1753-1828] had the same problem with his famous *History of English Birds*, which was published in 1797 and was widely known by the time Charlotte Brontë made a substantial reference to it in the opening chapter of *Jane Eyre*. Some of these woodcuts are images of particular species but others are whimsical scenes from the rural life in England at the time. Many have a humorous, or macabre tone. I have decided to use the blank space in this atlas to reproduce many of Bewick's woodcuts.



$$16.06 = D_8 \times 2 = \langle A^4, B^2, C^2, BA = A^{-1}B \rangle$$

CC	1	2	3	4	5	6	7	8	9	10	
Size	1	1	2	2	2	1	1	2	2	2	$\mathfrak{K}$
$\chi_1$	1	1	1	1	1	1	1	1	1	1	G
$\chi_2$	1	1	1	-1	-1	1	1	1	-1	-1	1
$\chi_3$	1	1	-1	1	-1	1	1	-1	1	-1	2
$\chi_4$	1	1	-1	-1	1	1	1	-1	-1	1	3
$\chi_5$	2	-2	0	0	0	2	-2	0	0	0	4
$\chi_6$	1	1	1	1	1	-1	-1	-1	-1	-1	5
$\chi_7$	1	1	1	-1	-1	-1	-1	-1	1	1	6
$\chi_8$	1	1	-1	1	-1	-1	-1	1	-1	1	7
$\chi_9$	1	1	-1	-1	1	-1	-1	1	1	-1	8
$\chi_{10}$	2	-2	0	0	0	-2	2	0	0	0	9
order	1	2	4	2	2	2	2	4	2	2	

**Normal subgroups**

	Classes	H	G/H	$\cap$	
1	1 + 2 + 3 + 6 + 7 + 8	$4 \times 2$	2	1	
2	1 + 2 + 4 + 6 + 7 + 9	$2^3$	2	2	
3	1 + 2 + 5 + 6 + 7 + 10	$2^3$	2	3	
4	1 + 6	2	$D_8$	4	
5	1 + 2 + 3 + 4 + 5	$D_8$	2	5	
6	1 + 2 + 3 + 9 + 10	$D_8$	2	6	
7	1 + 2 + 4 + 8 + 10	$D_8$	2	7	
8	1 + 2 + 5 + 8 + 9	$D_8$	2	8	
9	1 + 7	2	$D_8$	9	
10	1 + 2 + 6 + 7	$2^2$	$2^2$	$1 \cap 2$	$Z = \mathfrak{K}$
11	1 + 2 + 3	4	$2^2$	$1 \cap 5$	
12	1 + 2 + 8	4	4	$1 \cap 7$	
13	1 + 2 + 4	$2^2$	$2^2$	$2 \cap 5$	
14	1 + 2 + 9	$2^2$	$2^2$	$2 \cap 6$	
15	1 + 2 + 5	$2^2$	$2^2$	$3 \cap 5$	
16	1 + 2 + 10	$2^2$	$2^2$	$3 \cap 7$	
17	1 + 2	2	$2^3$	$8 \cap 10$	$G' = \Phi$

**Maximal subgroups:**  $[4 \times 2]$ ,  $[2^3] \times 2$ ,  $[D_8] \times 4$

**16.07 =  $\langle A^4, B^2 = A^2, C^2, BA = A^{-1}B \rangle = Q_8 \times 2$  Hamiltonian group**

CC	1	2	3	4	5	6	7	8	9	10	
size	1	1	2	2	2	1	1	2	2	2	$\mathcal{K}$
$\chi_1$	1	1	1	1	1	1	1	1	1	1	G
$\chi_2$	1	1	1	-1	-1	1	1	1	-1	-1	1
$\chi_3$	1	1	-1	1	-1	1	1	-1	1	-1	2
$\chi_4$	1	1	-1	-1	1	1	1	-1	-1	1	3
$\chi_5$	2	-2	0	0	0	2	-2	0	0	0	4
$\chi_6$	1	1	1	1	1	-1	-1	-1	-1	-1	5
$\chi_7$	1	1	1	-1	-1	-1	-1	-1	1	1	6
$\chi_8$	1	1	-1	1	-1	-1	-1	1	-1	1	7
$\chi_9$	1	1	-1	-1	1	-1	-1	1	1	-1	8
$\chi_{10}$	2	-2	0	0	0	-2	2	0	0	0	9
order	1	2	4	4	4	2	2	4	4	4	

**Normal subgroups**

	Classes	H	G/H	$\cap$	$\mathcal{K} = G$
1	1 + 2 + 3 + 6 + 7 + 8	$4 \times 2$	2	1	
2	1 + 2 + 4 + 6 + 7 + 9	$4 \times 2$	2	2	
3	1 + 2 + 5 + 6 + 7 + 10	$4 \times 2$	2	3	
4	1 + 6	2	$Q_8$	4	
5	1 + 2 + 3 + 4 + 5	$Q_8$	2	5	
6	1 + 2 + 3 + 9 + 10	$Q_8$	2	6	
7	1 + 2 + 4 + 8 + 10	$Q_8$	2	7	
8	1 + 2 + 5 + 8 + 9	$Q_8$	2	8	
9	1 + 7	2	$Q_8$	9	
10	1 + 2 + 6 + 7	$2^2$	$2^2$	$1 \cap 2$	
11	1 + 2 + 3	4	$2^2$	$1 \cap 5$	
12	1 + 2 + 8	4	$2^2$	$1 \cap 7$	
13	1 + 2 + 4	4	$2^2$	$2 \cap 5$	
14	1 + 2 + 9	4	$2^2$	$2 \cap 6$	
15	1 + 2 + 5	$2^2$	$2^2$	$3 \cap 5$	
16	1 + 2 + 10	$2^2$	$2^2$	$3 \cap 7$	
17	1 + 2	2	$2^3$	$8 \cap 10$	

**Maximal subgroups:**  $[4 \times 2] \times 3, [Q_8] \times 4$

$$16.08 = \langle A^4, B^2, C^2, CB = A^2BC \rangle$$

CC	1	2	3	4	5	6	7	8	9	10
elts	1	A <sup>2</sup>	A	A <sup>3</sup>	B A <sup>2</sup> B	AB A <sup>3</sup> B	C A <sup>2</sup> C	AC A <sup>3</sup> C	BC A <sup>2</sup> BC	ABC A <sup>3</sup> BC

CC	1	2	3	4	5	6	7	8	9	10	
size	1	1	1	1	2	2	2	2	2	2	$\mathfrak{K}$
$\chi_1$	1	1	1	1	1	1	1	1	1	1	G
$\chi_2$	1	1	1	1	1	-1	1	-1	-1	-1	1
$\chi_3$	1	1	1	1	-1	1	-1	1	-1	-1	2
$\chi_4$	1	1	1	1	-1	-1	-1	-1	1	1	3
$\chi_5$	1	1	-1	-1	1	1	-1	-1	1	-1	4
$\chi_6$	1	1	-1	-1	1	-1	-1	1	-1	1	5
$\chi_7$	1	1	-1	-1	-1	1	1	-1	-1	1	6
$\chi_8$	1	1	-1	-1	-1	-1	1	1	1	-1	7
$\chi_9$	2	-2	2i	-2i	0	0	0	0	0	0	0
$\chi_{10}$	2	-2	-2i	2i	0	0	0	0	0	0	0
order	1	2	4	4	2	4	2	4	4	2	

### Normal subgroups

	Classes	H	G/H	$\cap$
1	1 + 2 + 3 + 4 + 9 + 10	$4 \times 2$	2	1
2	1 + 2 + 5 + 6 + 9	$4 \times 2$	2	2
3	1 + 2 + 7 + 8 + 9	$4 \times 2$	2	3
4	1 + 2 + 3 + 4 + 5 + 7	$D_8$	2	4
5	1 + 2 + 3 + 4 + 6 + 8	$D_8$	2	5
6	1 + 2 + 5 + 8 + 10	$D_8$	2	6
7	1 + 2 + 6 + 7 + 10	$D_8$	2	7
8	1 + 2 + 9	4	$2^2$	$1 \cap 2$
9	1 + 2 + 3 + 4	4	$2^2$	$1 \cap 4$ $Z = \mathfrak{K}$
10	1 + 2 + 10	$2^2$	$2^2$	$1 \cap 6$
11	1 + 2 + 6	4	$2^2$	$2 \cap 4$
12	1 + 2 + 5	$2^2$	$2^2$	$2 \cap 6$
13	1 + 2 + 7	$2^2$	$2^2$	$3 \cap 4$
14	1 + 2 + 8	4	$2^2$	$3 \cap 5$
15	1 + 2	2	$2^3$	$8 \cap 9$ $G' = \Phi$

Maximal subgroups:  $[4 \times 2] \times 3, [D_8] \times 4$

$$16.09 = \langle A^4, B^2, C^2, CA = ABC \rangle$$

CC	1	2	3	4	5	6	7	8	9	10
elts	1	B	A <sup>2</sup>	A <sup>2</sup> B	C	A <sup>2</sup> C	A	A <sup>3</sup>	AC	A <sup>3</sup> C
					BC	A <sup>2</sup> BC	AB	A <sup>3</sup> B	ABC	A <sup>3</sup> BC

CC	1	2	3	4	5	6	7	8	9	10	
size	1	1	1	1	2	2	2	2	2	2	$\mathfrak{K}$
$\chi_1$	1	1	1	1	1	1	1	1	1	1	G
$\chi_2$	1	1	1	1	1	1	-1	-1	-1	-1	1
$\chi_3$	1	1	1	1	-1	-1	1	1	-1	-1	2
$\chi_4$	1	1	1	1	-1	-1	-1	-1	1	1	3
$\chi_5$	1	1	-1	-1	1	-1	i	-i	1	-i	4
$\chi_6$	1	1	-1	-1	1	-1	-i	i	-i	i	4
$\chi_7$	1	1	-1	-1	-1	1	i	-i	-i	i	5
$\chi_8$	1	1	-1	-1	-1	1	-i	i	i	-i	5
$\chi_9$	2	-2	2	-2	0	0	0	0	0	0	6
$\chi_{10}$	2	-2	-2	2	0	0	0	0	0	0	7
order	1	2	2	2	2	2	4	4	4	4	

**Normal subgroups**

	Classes	H	G/H	$\cap$	
1	1 + 2 + 3 + 4 + 5 + 6	$2^3$	<b>2</b>	1	
2	1 + 2 + 3 + 4 + 7 + 8	$4 \times 2$	<b>2</b>	2	
3	1 + 2 + 3 + 4 + 9 + 10	$4 \times 2$	<b>2</b>	3	
4	1 + 2 + 5	$2^2$	<b>4</b>	4	
5	1 + 2 + 6	$2^2$	<b>4</b>	5	
6	1 + 3	<b>2</b>	$D_8$	6	
7	1 + 4	<b>2</b>	$D_8$	7	
8	1 + 2 + 3 + 4	$2^2$	$2^2$	$1 \cap 2$	$Z = \Phi = \mathfrak{K}$
9	1 + 2	<b>2</b>	$4 \times 2$	$2 \cap 4$	$G'$

**Maximal subgroups:**  $[2^3], [4 \times 2] \times 2$

$$16.10 = D_{4,4} = \langle A^4, B^4, BA = A^{-1}B \rangle$$

CC	1	2	3	4	5	6	7	8	9	10
elts	1	$A^2$	$B^2$	$A^2B^2$	$A$ $A^3$	$AB^2$ $A^3B^2$	$B$ $A^2B$	$B^3$ $A^2B^3$	$AB$ $A^3B$	$AB^3$ $A^3B^3$

CC	1	2	3	4	5	6	7	8	9	10	
size	1	1	1	1	2	2	2	2	2	2	$\mathcal{K}$
$\chi_1$	1	1	1	1	1	1	1	1	1	1	G
$\chi_2$	1	1	1	1	1	1	-1	-1	-1	-1	1
$\chi_3$	1	1	1	1	-1	-1	1	1	-1	-1	2
$\chi_4$	1	1	1	1	-1	-1	-1	-1	1	1	3
$\chi_5$	1	1	-1	-1	1	-1	i	-i	i	-i	4
$\chi_6$	1	1	-1	-1	1	-1	-i	i	-i	i	4
$\chi_7$	1	1	-1	-1	-1	1	i	-i	-i	i	5
$\chi_8$	1	1	-1	-1	-1	1	-i	i	i	-i	5
$\chi_9$	2	-2	2	-2	0	0	0	0	0	0	6
$\chi_{10}$	2	-2	-2	2	0	0	0	0	0	0	7
order	1	2	2	2	4	4	4	4	4	4	

### Normal subgroups

Classes	H	G/H	$\cap$	
1	1 + 2 + 3 + 4 + 5 + 6	$4 \times 2$	2	1
2	1 + 2 + 3 + 4 + 7 + 8	$4 \times 2$	2	2
3	1 + 2 + 3 + 4 + 9 + 10	$4 \times 2$	2	3
4	1 + 2 + 5	$2^2$	4	4
5	1 + 2 + 6	$2^2$	4	5
6	1 + 3	2	$D_8$	6
7	1 + 4	2	$Q_8$	7
8	1 + 2 + 3 + 4	$2^2$	$2^2$	$1 \cap 2 \quad Z = \Phi = \mathcal{O}$
9	1 + 2	2	$4 \times 2$	$2 \cap 4 \quad G'$

Maximal subgroups:  $[4 \times 2] \times 3$

$$16.11 = M_{8,2}^{(5)} = \langle A^8, B^2, BA = A^5B \rangle$$

CC	1	2	3	4	5	6	7	8	9	10
elts	1	$A^4$	$A^2$	$A^6$	$A$ $A^5$	$A^3$ $A^7$	$B$ $A^4B$	$A^2B$ $A^6B$	$A^3B$ $A^7B$	$AB$ $A^5B$

CC	1	2	3	4	5	6	7	8	9	10	
size	1	1	1	1	2	2	2	2	2	2	$\mathfrak{K}$
$\chi_1$	1	1	1	1	1	1	1	1	1	1	G
$\chi_2$	1	1	1	1	1	1	-1	-1	-1	-1	1
$\chi_3$	1	1	1	1	-1	-1	1	1	-1	-1	2
$\chi_4$	1	1	1	1	-1	-1	-1	-1	1	1	3
$\chi_5$	1	1	-1	-1	I	-i	1	-1	-i	i	4
$\chi_6$	1	1	-1	-1	-i	i	1	-1	i	-i	4
$\chi_7$	1	1	-1	-1	I	-i	-1	1	i	-i	5
$\chi_8$	1	1	-1	-1	-i	i	-1	1	-i	i	5
$\chi_9$	2	-2	2i	-2i	0	0	0	0	0	0	0
$\chi_{10}$	2	-2	-2i	2i	0	0	0	0	0	0	0
order	1	2	4	4	8	8	2	4	8	8	

### Normal subgps

	Classes	H	G/H	$\cap$	
1	1 + 2 + 3 + 4 + 5 + 6	8	2	1	
2	1 + 2 + 3 + 4 + 7 + 8	$4 \times 2$	2	2	$\mathfrak{K}$
3	1 + 2 + 3 + 4 + 9 + 10	8	2	3	
4	1 + 2 + 7	$2^2$	4	4	
5	1 + 2 + 8	4	4	5	
6	1 + 2 + 3 + 4	4	$2^2$	$1 \cap 2$	$Z = \Phi$
7	1 + 2	2	$4 \times 2$	$1 \cap 4$	$G'$

Maximal subgroups:  $[8] \times 2$ ,  $[4 \times 2]$

$$16.12 = D_{16} = \langle A^8, B^2, BA = A^{-1}B \rangle$$

CC	1	2	3	4	5	6	7
elts	1	A <sup>4</sup>	A <sup>2</sup> A <sup>6</sup>	A A <sup>7</sup>	A <sup>3</sup> A <sup>5</sup>	B A <sup>2</sup> B A <sup>4</sup> B A <sup>6</sup> B	AB A <sup>3</sup> B A <sup>5</sup> B A <sup>7</sup> B

CC	1	2	3	4	5	6	7	
size	1	1	2	2	2	4	4	$\mathcal{K}$
$\chi_1$	1	1	1	1	1	1	1	G
$\chi_2$	1	1	1	1	1	-1	-1	1
$\chi_3$	1	1	1	-1	-1	1	-1	2
$\chi_4$	1	1	1	-1	-1	-1	1	3
$\chi_5$	2	2	-2	0	0	0	0	4
$\chi_6$	2	-2	0	$\sqrt{2}$	$-\sqrt{2}$	0	0	0
$\chi_7$	2	-2	0	$-\sqrt{2}$	$\sqrt{2}$	0	0	0
order	1	2	4	8	8	2	2	

### Normal subgroups

Classes	H	G/H	$\cap$	
1	1 + 2 + 3 + 4 + 5	8	2	1
2	1 + 2 + 3 + 6	$D_8$	$2^2$	2
3	1 + 2 + 3 + 7	$D_8$	4	3
4	1 + 2	2	$D_8$	4 <b>Z</b>
5	1 + 2 + 3	4	$2^2$	$1 \cap 2$ <b>G' = Z<sub>2</sub> = <math>\Phi</math> = <math>\mathcal{K}</math></b>

Maximal subgroups: [8]



$$16.13 = M_{8,2}^{(3)} = \langle A^8, B^2, BA = A^3B \rangle$$

	CC	1	2	3	4	5	6	7	
Eelts		1	A <sup>4</sup>	A <sup>2</sup> A <sup>6</sup>	A A <sup>3</sup>	A <sup>5</sup> A <sup>7</sup>	B A <sup>2</sup> B A <sup>4</sup> B A <sup>6</sup> B	AB A <sup>3</sup> B A <sup>5</sup> B A <sup>7</sup> B	

	CC	1	2	3	4	5	6	7	
size		1	1	2	2	2	4	4	$\mathcal{K}$
$\chi_1$		1	1	1	1	1	1	1	G
$\chi_2$		1	1	1	1	1	-1	-1	1
$\chi_3$		1	1	1	-1	-1	1	-1	2
$\chi_4$		1	1	1	-1	-1	-1	1	3
$\chi_5$		2	2	-2	0	0	0	0	4
$\chi_6$		2	-2	0	$\sqrt{2}i$	$-\sqrt{2}i$	0	0	0
$\chi_7$		2	-2	0	$-\sqrt{2}i$	$\sqrt{2}i$	0	0	0
order		1	2	4	8	8	2	4	

### Normal subgroups

Classes	H	G/H	$\cap$	
1	1 + 2 + 3 + 4 + 5	<b>8</b>	<b>2</b>	1
2	1 + 2 + 3 + 6	<b>D<sub>8</sub></b>	<b>2</b>	2
3	1 + 2 + 3 + 7	<b>Q<sub>8</sub></b>	<b>2</b>	3
4	1 + 2	<b>2</b>	<b>D<sub>8</sub></b>	4 <b>Z = <math>\mathcal{K}</math></b>
5	1 + 2 + 3	<b>4</b>	<b>2<sup>2</sup></b>	1 $\cap$ 2 <b>G' = Z<sub>2</sub> = <math>\Phi</math></b>

Maximal subgroups: [8], [D<sub>8</sub>], [Q<sub>8</sub>]



$$16.14 = Q_{16} = \langle A^8, B^2 = A^4, BA = A^{-1}B \rangle$$

CC	1	2	3	4	5	6	7
elts	1	A <sup>4</sup>	A <sup>2</sup> A <sup>6</sup>	A A <sup>7</sup>	A <sup>3</sup> A <sup>5</sup>	B A <sup>2</sup> B A <sup>4</sup> B A <sup>6</sup> B	AB A <sup>3</sup> B A <sup>5</sup> B A <sup>7</sup> B

CC	1	2	3	4	5	6	7	
size	1	1	2	2	2	4	4	$\mathcal{K}$
$\chi_1$	1	1	1	1	1	1	1	G
$\chi_2$	1	1	1	1	1	-1	-1	1
$\chi_3$	1	1	1	-1	-1	1	-1	2
$\chi_4$	1	1	1	-1	-1	-1	1	3
$\chi_5$	2	2	-2	0	0	0	0	4
$\chi_6$	2	-2	0	$\sqrt{2}$	$-\sqrt{2}$	0	0	0
$\chi_7$	2	-2	0	$-\sqrt{2}$	$\sqrt{2}$	0	0	0
order	1	2	4	8	8	4	4	

**Normal subgroups**

Classes	H	G/H	$\cap$	
1	1 + 2 + 3 + 4 + 5	8	2	1
2	1 + 2 + 3 + 6	Q <sub>8</sub>	2	2
3	1 + 2 + 3 + 7	Q <sub>8</sub>	2	3
4	1 + 2	2	D <sub>8</sub>	4 Z
5	1 + 2 + 3	4	2 <sup>2</sup>	1 $\cap$ 2 G' = $\Phi$ = $\mathcal{K}$

**Maximal subgroups:** [8], [Q<sub>8</sub>] × 2

